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senting the fundamental propositions and principles of mathematics in such constructive and developmental form as to *make each student feel the joy of individual discovery and creation.* The address contained some specific suggestions as to the subject matter and method of presentation of mathematics to meet these new ideals.

Professor Stager emphasized the need of greater coöperation between the university and the secondary school. This can be shown by a sympathetic interest on the part of the former in the varied, and often difficult, problems of the latter. High standards should be set by the university and should be insisted upon. Helpful suggestions should be offered through visitation of university instructors to the schools and teachers' meetings, and through carefully prepared bulletins. The university entrance requirements should receive re-adjustment: (1) by defining the two years of required mathematics as *elementary mathematics,* rather than as *algebra and plane geometry,*[1] which would permit of greater efficiency in teaching and afford those teachers favorable to the idea of so-called "unified mathematics" an opportunity to work out their ideas; (2) by allowing the second year of algebra to be devoted, one half to the more important, but less theoretical portions, of the present requirement, and the other half to the elementary fundamental principles of analytical geometry, more especially of the straight line and the circle. This would prove equally advantageous to both the university and the secondary school.

The two sessions were each attended by about seventy-five teachers and the papers were followed by spirited discussions.

BOOK REVIEWS.

Edited by W. H. Bussey, University of Minnesota.

Elementary Mathematical Analysis. A text-book for first year college students. By Charles S. Slichter. McGraw-Hill Book Company, New York, 1914. xiv + 490 pages.

Note.—The editors asked two men representing quite different phases of mathematical interests to review this book, and they take pleasure in presenting to the readers of the Monthly these two reviews.

I.

For years it has been assumed that mathematics should be administered in definite doses. After a pupil has emerged from the grammar school he is to be treated to one year of elementary algebra, one of plane geometry, then a half year each, of solid geometry, plane trigonometry, intermediate algebra, and advanced algebra. The tendency has been to prescribe the table of contents of each of these subjects. A student who can pass examinations in them all is ready to commence the study of analytic geometry, for which considerable familiarity with graphical methods can be presupposed from the algebra. But in many cases the preparation in algebra has been found unsatisfactory, so that now attempts are being made to accept alternate subjects for college entrance and to provide for instruction in advanced algebra in college. In the book under review a much greater step is taken; only elementary algebra and plane geometry are presupposed; the volume attempts to provide instruction in trigonometry, intermediate and advanced algebra, and in plane analytic geometry. Use is made of solid geometry but once (p. 416).

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[1] This provision was made by the University of Chicago two years ago. Editor.

To make such a programme economical, the school algebra and geometry should be given during the last two years of high-school rather than during the first two years as at present, otherwise the student would commence his college mathematics in a very rusty condition. Partial provision is made for this state of affairs, however, by including a sixteen page review of elementary algebra in the volume.

The table of contents gives only the names of the chapters, some of which contain several pages of material not connected with the title. This lack is partly compensated for by a full index. Several hundred exercises for the student are included, the answers to which are not furnished.

The first and second chapters are largely descriptive; they deal with the ideas of scale, functionality, and of graphical representation, with particular attention to power functions. The presentation is enlivened by a large number of illustrations of practical applications. The next chapter is a successful combination of the elements of trigonometry and analytic geometry of the circle. The trigonometric functions are at once defined for the general angle, thus making unnecessary much of the explanatory matter found in the ordinary texts. Indeed, this scheme could have been applied to a number of particular cases, where the author prefers to employ the ordinary right-angle triangle. It is to be regretted that the same generality was not always preserved in the later chapter on the addition formulas.

It seems odd to find in the chapter entitled "Ellipse and Hyperbola" the graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$, and proofs of the theorems: limit $x/\sin x = 1$, limit $x/\tan x = 1$. The chapter on simultaneous equations gives the factor theorem and a number of graphical illustrations, but nothing more from the theory of equations. Synthetic division and Horner's method are no where mentioned.

Next follows a chapter on permutations and combinations, the main use of which is to prove the binomial theorem; the next chapter, on progressions, is provided with a number of graphical illustrations. The substance of these three chapters is usually included in elementary algebra.

The longest chapter (68 pages) and perhaps the best chapter is entitled "The logarithmic and trigonometric functions." It begins with a historical outline and accompanies it with graphical illustrations, making the development natural and interesting. After the elementary properties have been established, they are applied to explain the double scale, the slide rule, uses of logarithmic paper and the interpretation of data gathered from a number of engineering problems.

Chapter IX includes the machinery of trigonometry, the solution of triangles, the graph of a power of sin x, etc.

Another commendable chapter is the tenth, on waves; it discusses the meaning of the terms used and shows how to apply the analysis in a large variety of ways. The chapter on complex numbers seems unnecessarily long (40 pages). It seems to the reviewer that all the essential facts could be stated in much less space.

The chapter on loci is concerned almost entirely with problems. They include several on the lemniscate, the cycloidal curves, and various others. The chapter on the conic sections contains the traditional treatment of many of the problems of analytic geometry, including tangents and normals, the general equation of a conic, and confocal conics. The criterion given for determining the positive and negative sides of a line is not applicable to lines which pass through the origin.

No mention is made of determinants. The idea of a derivative is introduced to obtain the slope of a general parabola, and of the exponential curve. The first case is easily justified, but the second is questionable, at first sight.

Whether this programme is a wise one can be determined only from experience. A priori praise or blame has but little meaning; at any rate it is an earnest effort to remove some of the difficulties of our present schedules.

VIRGIL SNYDER.

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II.

The first and most permanent impression which Slichter's Elementary Analysis makes upon one is of originality, of extraordinary and all-pervasive originality. The work is so different from any I have taught or any I know that I should be unwilling to give any estimate of its usability as a class text. Better than a review would be a series of statements by a number of persons who had used the book. And in this connection I might suggest that the MONTHLY could do no greater service to collegiate teaching and teachers of mathematics than by replacing or supplementing its "Book Reviews" by conducting an experience meeting on texts which have had a fairly wide adoption.

When we review an ordinary book we have only to compare it with others of its like and to make some rather inaccurate estimate of how it would impress a class. When we have to deal with an iconoclastic work we must go back to first principles, and these principles have not yet been definitely established, except in so far as the historical development of mathematical teaching to the present time may be considered to have established them, and this test is always unfair to the original work.

We shall therefore ask first the question: What should be taught in the freshman year? The canonical answer is, algebra, trigonometry, and analytic geometry, in various proportions. Now analytic geometry, except for that part which introduces the student to coördinates and elementary curve tracing and which may better be designated as graphical algebra, is a purely mathematical subject which is distinctly difficult for a student to assimilate in his most immature collegiate year. The real problem of analytic geometry is to obtain geometrical results by algebraic means, and experience seems to show that this is too much for the freshman; moreover, it is beyond the needs of any but the student of pure mathematics or of really advanced applications. There has sprung up a strong tendency toward the elimination of true analytics from freshman work, and we hope the tendency will become a fixed habit. Slichter pays scant attention to analytics.

The process of attrition has also been applied to trigonometry. There are many old texts built upon the practice of nearly, if not quite, a whole year's work devoted to the subject. Undoubtedly plenty of material of importance exists from which a long course in trigonometry can be constructed, but not without calling upon many items which are unnecessary for the ordinary student. We are therefore getting down to twenty or thirty lessons on trigonometry, and Slichter has no more than this (at least of the canonical sort of trigonometry).

When our students come to college they have been over a considerable amount of algebra, probably as much as, or more than, most of them will need; but they frequently do not know how to use rapidly and accurately even the smaller amount which they must meet almost daily. A review of secondary school algebra would be a considerable portion of the first few weeks work in the freshman year if we put our students' efficiency before our academic pride. Slichter closes his book with such a review and with the statement that the review should precede Chapter I. (Why not print it there?) He does not include simultaneous quadratic equations in this review; they are treated appropriately but very inadequately, especially as regards exercises for the student to do, after the common second order curves have been presented. As for additional algebra, the author gives us permutations and combinations, the binomial theorem, and progressions, interesting material briefly treated.

We see therefore that as regards algebra, trigonometry, and analytic geometry we are on safe modern ground in retaining the essentials and in eliminating the more advanced parts.

The treatment of these three subjects is according to the mixed method. We do not proceed with one until it is completed but oscillate from one to another. The first two chapters are analytics (used, as always in this review, for graphical algebra or curve tracing), the third is trigonometry, the fourth returns to curve tracing and the fifth to its more algebraic side (solution of equations). Chapters VI and VII are algebraic, Chapter VIII is on logarithms and exponentials, and Chapter IX is on trigonometry again. The tenth and eleventh chapters on waves and complex numbers (with polar diagrams of periodic functions) are for the most part curve tracing once more, while the last two chapters on loci and the conic sections approach true analytics without quite reaching it.

It will be noticed that the author has really welded his material together; he does not switch violently from one subject to another just for the sake of pursuing the mixed method; there are modulations. Ordinarily it would not be possible for the teacher to transpose the chapters and rearrange the work according to subjects—though I am by no means certain that such a transposition could not be made with Chapter IX so that it might follow Chapter III.

It is now time for a second fundamental question: Is the mixed method good? We all know the difficulty it is supposed to avoid, namely, the isolation of the student's knowledge into mutually insulated bodies. Some also feel that by not classifying a student's knowledge you give him greater power, you lead him to apply all his mathematical equipment to a new problem, instead of some particular part of his equipment such as trigonometry. The mixed method as contrasted with the compartment method of instruction is so young that we cannot yet say whether it is better or worse. There is the possibility, which at times appears to me as a strong probability, that by introducing broad connecting and conducting links between the different compartments you so reduce the potential that the efficiency is lowered. May it not be true that most problems which the majority of us solve are really problems in algebra, in trigonometry, or in analytics, and that the training in classification according to these topics is highly valuable? Is not the mixed method mixing to the student, and may not the oscillation be allied to vacillation? Only experience can answer.

As the author has reduced the ordinary subjects to such moderate compass, what has he done with the rest of his five hundred pages? He has introduced a large number of topics not usually found in a freshman text, perhaps not in any mathematical text used by mathematicians—double and triple scales, the slide-rule, plotting on logarithmic and semi-logarithmic and sinusoidal coördinate paper, simple harmonic motion, damped vibrations, logarithmic decrement, waves, shearing motion, graphs of many engineering formulas, and plenty of odds and ends. In these and in many other ways the work suggests Perry's "Calculus for Engineers."

It is true that all the things which the author takes up may logically be taken up; but we are brought back to the first question: What ought to go into the freshman course? For myself I do not believe in a mathematical or engineering museum for first year students. When a book starts in to give a series of illustrations and appends a footnote (as on page 68) stating: "The instructor is expected fully to explain the meaning of the technical terms here used," I get out of sorts with the author. The freshman is an immature person with a limited range of ideas and with a great many ideas in this limited range in need of clarification; the fewer unfamiliar terms that are introduced in his instruction the better, and the greater the chance of impressing on him the main thing in mathematical instruction, to wit, to think clearly.

Among the ideas which every freshman has, but has only vaguely, is the idea of speed. The word speed does not occur in Slichter's index, nor does velocity except for (uniform) angular velocity and velocity of a wave. The word rate is also missing. One may remark that without entering upon the notions of calculus these words cannot be defined. Precisely, and that is the fundamental reason for entering as promptly as possible upon those ideas. The author urges that one of his chief aims is to impress the concept of functionality on the student. He does impress it so far as graphical algebra alone can impress it; but of prime importance in the idea of functionality is that of a rate of change, and into this he does not go except for the mention once or twice of slope.

Here is the really immoral element in the book: The author has cut down the algebra, the trigonometry, and the analytics to a point where the student has not acquired a real facility of operation with any of the three, and then instead of turning the time and space thus saved into a presentation of the elements of the calculus, he has frittered both away with frills which are more or less unintelligible to the student and of incomparably less value to him if understood.

There are three objects in mathematical instruction: (1) To give the student a training in formal manipulation which will suffice him for his ordinary needs—and to give this it is necessary to take him over considerably more than what he will actually use, just as to play satisfactorily a sonata one needs a technique developed beyond that of the piece; one must have reserve power. (2) To formulate for the student and to drill into him the fundamental ideas which he meets daily and with which in many cases he is already vaguely familiar—speed, rates, limits of sums—and here again he should go far enough to have reserve power, whereas most texts on calculus do not take him far enough so that he can formulate a definition for flux or induction. (3) To give him a taste of the applications of his ideas and his technique in mechanics or physics—and this could be done by the departments of mechanics and physics if they were not so terribly chary of using the mathematics that the student has (possibly the teacher has it no longer?).

Now it is axiomatic that you cannot teach all these subjects in a single year, and to freshmen at that. In particular (3) must be almost entirely omitted at first. You cannot even give a freshman both his technique and his ideas. In this dilemma the preference used to be given to qtechniue, and with a good reason; for the young will unhesitatingly accept a formal method and, being imitative rather than rational, will rapidly acquire all the necessary facility. If this preference were still to be followed, we should give the freshman a thorough course in trigonometry, including trigonometric equations and identities, which are insufficiently treated by our author, and perhaps a few applications to statics in the plane; we should follow with a small amount of curve tracing, and then finish the formal differential calculus including a few geometric and kinematic applications. The sophomore year would begin with formal integration.

The serious objection of a practical nature to this procedure is that the ideas of the calculus do not come early enough to be most useful in the work in physics. We ought therefore to turn to the other preference and give the freshman a brief course in trigonometry (which might better be given in the secondary schools), a little curve tracing, and the ideas of the differential and integral calculus, keeping the formal work so simple that it will offer no difficulty. The sophomore year would then begin with trigonometric identities and drill in differentiation and integration. The technique will not come too late for the work in physics and engineering, because not much technique is required for the early instruction in these subjects.

We have gone into these matters in such detail because we have had to go back to fundamentals and because we believe that the author has made the worst possible use of his extra space and time. We shall now try to give some justification for our feeling that the book is intensely original. The ellipse is defined as the locus obtained by shortening the ordinates of a circle in a definite ratio, and some two hundred pages later the tangent to the ellipse is found by applying the same rule to the ordinates of the tangent to the circle. The ellipse is given in parametric form, the loci $y = \tan x$ and $y = \sec x$ are plotted, and the hyperbola is defined and plotted from its parametric form $x = a \sec \theta$, $y = b \tan \theta$. The tangent to the hyperbola is not found. The tangent to the parabola is found by expressing the fact that it cuts the curve in coincident points; that to the circle, by its perpendicularity to the radius. The normal equation of a straight line and the distance from a point to a line occur near the end of the book in the chapter on conics, presumably because by means of them the passage from the tangent to a circle to the tangent to an ellipse is made.

At the very beginning of the book the student is told what drawing materials he should have and a considerable amount of graphical computation is given. This comes in handy in treating logarithms which are introduced by arranging arithmetic and geometric progressions in parallel columns. Exponential curves are constructed and the number e is defined as that value of r for which $y = r^x$ passes through (0, 1) with the slope 1. In this connection and only here is the slope related to the limit of the slope of a secant. Later e is found by a familiar operation on the binomial expansion, an operation which is justified by the (presumably untrue) statement that in the calculus it is shown that

$$\lim_{n=\infty}\left[1+1+\frac{1-\frac{1}{n}}{2!}+\frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)}{3!}+\cdots\right]=1+1+\frac{1}{2!}+\frac{1}{3!}+\cdots.$$

These illustrations, taken with earlier mention of unusual things discussed by the author, may indicate the originality of the work, an originality which grows on one constantly. Indeed it would be little exaggeration to say that if there was an unusual way of doing anything, the author has chosen that way and worked it down so smoothly that few who did not know would consider it new.

For one who lays such stress on calculation and gives such elaborate tables illustrating how an expert computer would arrange the solution of a triangle, it is surprising to find the author paying no attention to the vertical alignment of his decimal points in the table. I have not seen many examples of the work of expert computers, but what few I have seen look very different in this particular. Moreover, in solving a triangle with two sides and the included angle given, why use the tangent formulas? Why not be original and drop the perpendicular from the shorter side upon the longer? This solution may not be quite so neat, but is good enough, and saves the student the task of learning a new method.

The author is constantly and terribly mixed up in his statements and notations, though of course not in his ideas, about number and magnitude. Unless the reader can supply a great deal he cannot properly interpret the statements on pages 9, 43, 66, 343, and I fear that there are exercises (for the freshman) on this last page which I could not myself answer with any assurance of agreeing with the author.

Slichter's Elementary Mathematical Analysis should be widely tried out, if only for the rest that it will give the teacher from the familiar beaten paths; there is a charming freshness about the work and, whether we like it or not, it is bound to be ranked as a distinct contribution to the theory and practice of freshman instruction in mathematics.

E. B. WILSON.

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PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND R. P. BAKER.

PROBLEMS FOR SOLUTION.

ALGEBRA.

When this issue was made up, no solutions had been received for numbers 417–426.

426. Proposed by HERBERT N. CARLETON, West Newbury, Mass.

Find all the solutions of the equation $x\sqrt[x]{x} = x^x$.

427. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

If $r \sin (\theta + \alpha) = m$ and $r \cos (\theta + \beta) = n$, show that

$$r = \frac{\sqrt{m^2 + n^2 - 2mn \sin (\alpha - \beta)}}{\cos (\alpha - \beta)},$$

GEOMETRY.

When this issue was made up, no solutions had been received for numbers 447–8, 450–454.

455. Proposed by R. P. BAKER, University of Iowa.

Find the minimum triangle of assigned angles inscribed in a given triangle.

456. Proposed by J. W. CLAWSON, Ursinus College.

The interior and exterior bisectors of the angles A, B, C of a triangle meet the opposite sides in U, U'; V, V'; W, W' respectively. Circles are drawn on UU', VV', WW' as diameters (Circles of Apollonius.) Prove that (1) These three circles have a common chord. (2) The centre of the circumcircle lies on this common chord.

CALCULUS.

When this issue was made up, no solutions had been received for numbers 358, 361–2, 364–372, 374–5, and 377.

376. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that

$$\frac{1}{z} - \frac{1}{z}(1 - 2xz + z^2)^{\frac{1}{2}} = x + \frac{z}{2}\left(\frac{x^2 - 1}{1 - xz}\right) + \sum_{n=2}^{\infty} \frac{1, 3, 5 \cdots 2n - 3}{2, 4, 6 \cdots 2n}(x^2 - 1)^n \left(\frac{z}{1 - xz}\right)^{2n-1}$$